

# Mark Scheme (Results)

January 2017

Pearson Edexcel International Advanced Subsidiary Level In Core Mathematics C12 (WMA01) Paper 01



Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at <a href="https://www.edexcel.com">www.edexcel.com</a> or <a href="https://www.btec.co.uk">www.btec.co.uk</a>. Alternatively, you can get in touch with us using the details on our contact us page at <a href="https://www.edexcel.com/contactus">www.edexcel.com/contactus</a>.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2017
Publications Code WMA01\_01\_1701\_MS
All the material in this publication is copyright
© Pearson Education Ltd 2017

#### General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

#### PEARSON EDEXCEL I AL MATHEMATICS

#### General Instructions for Marking

- 1. The total number of marks for the paper is 125
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

#### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol √ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

#### Method mark for solving 3 term quadratic:

#### 1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where  $|pq| = |c|$ , leading to  $x = \dots$   
 $(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$ 

#### 2. Formula

Attempt to use the <u>correct</u> formula (with values for a, b and c).

#### 3. Completing the square

Solving 
$$x^2 + bx + c = 0$$
:  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$ 

#### Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1.  $(x^n \rightarrow x^{n-1})$ 

#### 2. Integration

Power of at least one term increased by 1.  $(x^n \rightarrow x^{n+1})$ 

#### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

#### **Exact answers**

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

#### **Answers without working**

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

Question Number		Scheme	Marks
1(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)\frac{3x^2}{3} - 2 \times 2x + 3$	M1: $x^n \rightarrow x^{n-1}$ or $5 \rightarrow 0$ A1: <b>Any 3</b> of the 4 terms differentiated correctly - this could be 2 terms correct and $5 \rightarrow 0$ (allow simplified or un-simplified for this mark, including $3x^0$ for 3)	M1A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = x^2 - 4x + 3$	Cao. All 3 terms correct and simplified and on the same line and no + 0. (Do not allow $1x^2$ for $x^2$ or $x^1$ for $x$ or $3x^0$ for 3). Condone poor notation e.g. omission of $dy/dx = \dots$ or if they use $y = \dots$	A1
		iply by 3 before differentiating:	
	e.g. $\left(\frac{x^3}{3} - 2x^2 + 3x + 5\right) \times 3 =$	$x^{3} - 6x^{2} + 9x + 15 \Rightarrow \frac{dy}{dx} = 3x^{2} - 12x + 9$	
	Scores M1A0A0 but could	recover in (a) if they then divide by 3	
	If they persist with $\frac{dy}{dx} = 3x^2 - \frac{dy}{dx}$	-12x+9 in (b), allow full recovery in (b)	
			(3)
(b)	$x^2 - 4x + 3 = 0 \Rightarrow x = 1, 3$	M1: Attempt to solve their 3TQ from part (a) as far as $x =$ (see general guidance for solving a 3TQ). If no working is shown and the roots are incorrect for their 3TQ, score M0 here but the second method mark below is still available.  A1: Correct values (may be implied by their inequalities e.g. a correct quadratic	M1A1
	fo	followed by just $x > 1$ and $x > 3$ could score M1A1 here)	
	<i>x</i> < "1", <i>x</i> > "3"	Chooses outside region $(x < \text{their lower limit } x > \text{their upper limit})$ . Do not award simply for diagram or table.	M1
	λ	x < 1,  x > 3	
	Correct answer. Allow the correct regions separated by a comma or		
	written separately and allow other notation e.g. $(-\infty,1)\cup(3,\infty)$ . Do not		
	allow $1 > x > 3$ or $x < 1$ and $x > 3$ (These score M1A0). ISW if possible		A1
	e.g. $x > 3$ , $x < 1$ followed by $1 > x > 3$ can score M1A1. $x > 3$ , $x > 1$ followed by $x > 3$ (or) $x < 1$ can score M1A1. Fully correct answer with no working scores both marks. Answers that are otherwise correct but use		
	working scores both marks. Answers that are otherwise correct but use $\leq$ , $\geq$ lose final mark as would $[-\infty,1] \cup [3,\infty]$ .		
			(4)
			(7 marks)

	Pnys	sicsAndMathsTutor.com	Т
Question Number		Scheme	Marks
2 (a)	Ma	rk (a) and (b) together	
	$(x\pm 4)(y\pm 2)$	Attempts to complete the square on $x$ and $y$ or sight of $(x\pm 4)$ and $(y\pm 2)$ . May be implied by a centre of $(\pm 4, \pm 2)$ . Or if considering $x^2 + y^2 + 2gx + 2fy + c = 0$ , centre is $(\pm g, \pm f)$ .	M1
	Centre $C = (4, -2)$	Correct centre (allow $x = 4$ , $y = -2$ ) But not $g =, f =$ or $p =, q =$ etc.	A1
	Correct	answer scores both marks	
			(2)
(b)	$r^2 = 12 + \left(\pm 4\right)^2 + \left(\pm 2\right)^2$	or $r = \sqrt{g^2 + f^2 - c}$ Must clearly be identifying the radius or radius <sup>2</sup> May be implied by a correct exact radius or awrt 5.66	M1
	$r = \sqrt{32}$	$r = \sqrt{32}$ . Accept exact equivalents such as $4\sqrt{2}$ . $r = \dots$ not needed but must clearly be the radius. Do not allow $\pm\sqrt{32}$ unless minus is rejected	A1
	Correct	answer scores both marks	
			(2)
(c)	$x = 0 \Longrightarrow y^2 + 4y - 12$	= 0 Correct quadratic. Allow $16+(y+2)^2 = 32$	B1
	$(y+6)(y-2) = 0 \Rightarrow y$	Attempts to solve a 3TQ that has come from substituting $x = 0$ or $y = 0$ into the given equation or their 'changed' equation. May be implied by correct answers for their quadratic.	M1
	y = 2, -6 or $(0, 2)$ and $($	Correct y values or correct coordinates. Accept sight of these for all 3 marks if no incorrect working seen but must clearly be y values or correct coordinates. This may be implied by the correct roots of a quadratic in y.	A1
			(3)
			(7 marks)

Question Number	Scheme		Marks
3(a)	$S = r\theta = 7 \times 0.8 = 5.6 \text{ (cm)}$	M1: Uses $S = r\theta$ A1: 5.6 oe e.g. 28/5	M1A1
	Note that if the 0.8 is converted to	degrees e.g. $0.8 \times \frac{180}{\pi} = 45.8366$ ,	
		or truncated when attempting M1 so allow A1 for awrt 5.6	
	300		(2)
(b)		M1: Attempts to find $\frac{\pi}{2} - 0.8$ or	
	$\angle POC = \frac{\pi}{2} - 0.8 = \text{awrt } 0.771$	$\pi - \frac{\pi}{2} - 0.8$ . Allow an attempt to	M1A1
	2 0.0 - 2 0.0 - 2 0.771	find $\theta$ from $\theta + \frac{\pi}{2} + 0.8 = \pi$ .	1411711
		Accept as evidence awrt 0.77 A1: awrt 0.771	
	Answers in degrees	only can score M1A0	
	e.g. 180-90-0.8	$\times \frac{180}{\pi} (= 44.163)$	
			(2)
(c)	$4^{2} + 5^{2} - 2 \times 4 \times 5 \cos'(0.771')$ or $\sqrt{4^{2} + 5^{2} - 2 \times 4 \times 5 \cos'(0.771')}$	Correct use of the cosine rule to find $CP$ or $CP^2$ . NB 0.771 radians is awrt 44 degrees. <b>Ignore lhs for this</b> mark and look for e.g. $4^2 + 5^2 - 2 \times 4 \times 5 \cos' 0.771$ or 44'	M1
	$CP^{2} = 4^{2} + 5^{2} - 2 \times 4 \times 5 \cos 0.771$ or $CP = \sqrt{4^{2} + 5^{2} - 2 \times 4 \times 5 \cos 0.771}$	A correct expression for <i>CP</i> or <i>CP</i> <sup>2</sup> with lhs consistent with rhs. Allow awrt 0.77 radians or awrt 44 degrees. (May be implied if a correct numerical value is used in subsequent work)	A1
	Perimeter = $4+5+2\times7+'5.6'+'3.5'$	$4+5+2\times7$ + their $AQ$ + their $CP$ . Need to see all 6 lengths but may be implied by e.g. $23+5.6+3.5$	M1
	= 32.11 (cm)	Awrt 32.11 (ignore units)	A1
			(4)
			(8 marks)

Question Number	Sch	neme	Marks
4 (a)	$S_9 = 54$ $\Rightarrow 54 = \frac{9}{2}(2a + 8d)$ or $\Rightarrow 54 = \frac{9}{2}(a + a + 8d)$	Uses a correct sum formula with $n = 9$ and $S_9 = 54$	M1
_	$\Rightarrow a+4d=6*$	cso	A1*
_		ting:	
		$+ \dots + a + 8d = 54$	
	$\Rightarrow 9a + 36a$	d = 54	
	Scores M1 for attempting to s	um 9 terms (both lines needed)	
	or $a+a+d+a+2d+a+3d+a+4d+a+5d+a+6d+a+7d+a+8d=54$ Scores M1 on its own and then A1 if they complete correctly.		
-			(2)
(b)	$a+7d = \frac{1}{2}(a+6d)$ or $\frac{1}{2}(a+7d) = a+6d$	Uses $t_8 = \frac{1}{2}t_7$ or $\frac{1}{2}t_8 = t_7$ to produce one of these equations.	M1
	$\Rightarrow 6 - 4d + 7d = \frac{1}{2} (6 - 4d + 6d)$ $\Rightarrow d = \dots$	Uses the <b>given equation from (a)</b> and their second linear <b>equation</b> in a and d and proceeds to find a value for either a or d.	M1
	$\Rightarrow d = -1.5, a = 12$	A1: Either $d = -1.5$ ( <i>oe</i> ) or $a = 12$ A1: Both $d = -1.5$ ( <i>oe</i> ) and $a = 12$	A1A1
	Note that use of $\frac{1}{2}t_8 = t_7$ in	(b) gives $a = 30$ and $d = -6$	
			(4)
		•	(6 marks)

Question Number	Schem	ne	Marks
5 (a)(i)	$\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9 = y - 2$	M1: $\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9$ or $\log_3\left(\frac{x}{9}\right) = \log_3 x + \log_3\frac{1}{9}$ Correct use of the subtraction rule or addition rule. Ignore the presence or absence of a base and any spurious "= 0"  A1: $y-2$	M1A1
	An answer left as $\log_3$	3 <sup>y-2</sup> scores M1A0	
	Note that $\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 x$		
(ii)	$\log_3 \sqrt{x} = \log_3 x^{\frac{1}{2}} = \frac{1}{2} \log_3 x = \frac{1}{2} y$	$\frac{1}{2}y$ or equivalent	B1
	3 2 3 2	2	(3)
(b)	$2\log_{3}\left(\frac{x}{9}\right) - \log_{3}\sqrt{x} = 2 \Rightarrow 2(y-2) - \frac{1}{2}y = 2$		(8)
	Uses their answers from part (a) to creat poor use of brackets e.g. $2(y-2)=2y-$ for this m	M1	
	$\Rightarrow y = 4$	Correct value for <i>y</i> .	A1
	Note that arriving at $(y-2)^2 - \frac{1}{2}y = 2$ ab		
	have a solution $y = 4$ so look out for y	Correct method for undoing log.	D.//1
	$\log_3 x = 4 \Rightarrow x = 3^4$	Dependent on the first M	dM1
	$\Rightarrow x = 81$	cao	A1 (4)
			(7 marks)
	$2\log_{3}\left(\frac{x}{9}\right) - \log_{3}\sqrt{x}$	$=\log_{3}\left(\frac{\left(x/9\right)^{2}}{\sqrt{x}}\right)$	
	or $2\log_3\left(\frac{x}{9}\right) - \log_3\sqrt{x} = 2\log_3 x - 2\log_3 9 - \log_3\sqrt{x} = \log_3\frac{x^2}{\sqrt{x}} + \dots$ Combines <b>two log terms in</b> <i>x</i> <b>correctly</b> to obtain a single log term		M1
Alt 1 (b)	$\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right) = 2$		
	$\log_3\left(\frac{x^2}{\sqrt{x}}\right) = 6$	Correct equation	A1
	$\left(\frac{\left(x/9\right)^2}{\sqrt{x}}\right) = 3^2 \text{ or } \left(\frac{x^2}{\sqrt{x}}\right) = 3^6$	Correct method for undoing log.  Dependent on the first M	<b>d</b> M1
	$\Rightarrow x = 81$	cao	A1

	$2\log_3\left(\frac{x}{9}\right) - \log_3\sqrt{x} = 2\log_3\left(\frac{3}{9}\right)$ Combines logs		M1
Alt 2 (b) Uses $x = 3^y$	$\log_3 \left( \frac{3^{\frac{3y}{2}}}{81} \right) = 2 \Rightarrow y = 4$	Correct value for y	A1
x=3	3 (81)	,	
	$\log_3 x = 4 \Longrightarrow x = 3^4$	Correct method for undoing log. <b>Dependent on the first M</b>	<b>d</b> M1
	$\Rightarrow x = 81$	cao	A1

Question Number	•	neme	Marks
6(a)(i)	$\frac{3}{2}$	Accept exact equivalents	B1
(ii)	$y = 0,  3x + 5 = 0 \Rightarrow x = -\frac{5}{3}$	M1: Sets $y = 0$ and attempts to find $x$ . Accept as evidence $3x+5=0 \Rightarrow x=$ or $awrt-1.7$ A1: $x=-\frac{5}{3}$ or exact equivalent including 1.6 recurring (i.e. a clear dot over the 6)	M1A1
(b)		1	(3)
	Gradient $l_2 = -\frac{1}{\frac{3}{2}} = -\frac{2}{3}$	Uses $m_2 = -\frac{1}{m_1}$ to find the gradient of $l_2$ (may be implied by their line equation). Allow an attempt to find $m_2$ from $m_1 \times m_2 = -1$ .	M1
	Point <i>B</i> has <i>y</i> coordinate of 4	This may be embedded within the equation of the line but must be seen in part (b).	B1
	e.g. $y-'4' = '-\frac{2}{3}'(x-1)$ or $\frac{y-'4'}{x-1} = '-\frac{2}{3}'$	A correct straight line method with a changed gradient and their point $(1, '4')$ . There must have been attempt to find the y coordinate of B. If using $y = mx + c$ , must reach as far as finding a value for $c$ .	M1
	e.g. $y-4 = -\frac{2}{3}(x-1)$ or $\frac{y-4}{x-1} = -\frac{2}{3}$	A correct un-simplified equation	A1
	2x+3y-14=0	Accept $A(2x+3y-14)=0$ where $A$ is an integer. Terms can be in any order but must have '= 0'.	A1
			(5)
Alt (b)	Gradient $l_2 = -\frac{1}{\frac{3}{2}} = -\frac{2}{3}$	Uses $m_2 = -\frac{1}{m_1}$ to find the gradient of $l_2$ as before	M1
	$\frac{3}{2}x + \frac{5}{2} = -\frac{2}{3}x + c$	A correct statement for $l_1 = l_2$	B1
	$x = 1 \Rightarrow c = \frac{14}{3}$ $y = -\frac{2}{3}x + \frac{14}{3}$	Substitutes $x = 1$ to find a value for $c$	M1
	$y = -\frac{2}{3}x + \frac{14}{3}$	Correct equation	A1
	2x+3y-14=0	Accept $A(2x+3y-14) = 0$ where A is an integer.	A1

PhysicsAndMathsTutor.com				
(c)	$y = 0 \Rightarrow 2x - 14 = 0 \Rightarrow x = 7$	Attempts to find $C$ using $y = 0$ in the equation obtained in part (b)	M1	
	Attempts Area of triangle us	Attempts Area of triangle using $\frac{1}{2} \times AC \times (y \text{ coord of } B)$		
	$=\frac{1}{2}\times\left('7'+'\frac{5}{3}'\right)\times'4'$			
	o. Attempts Area of trian		M1	
	$\frac{1}{2} \times \left(1 + \left(\frac{5}{3}\right)^{1}\right) \times \left(y \text{ coord of } B\right)$	$+\frac{1}{2}\times('7'-1)\times(y \text{ coord of } B)$		
	If they make a second/different attem still allow	-		
	$=\frac{52}{3}$	Area = $\frac{52}{3}$ or exact equivalent e.g	A1	
	$-{3}$	$17\frac{1}{3}$ or 17.3 recurring (i.e. a clear dot over the 3)	Al	
		,	(3)	
			(11 marks)	
	$y = 0 \Rightarrow 2x - 14 = 0 \Rightarrow x = 7$	Attempts to find $C$ using $y = 0$ in the equation obtained in part (b)	M1	
Way 2	Attempts area of triangle using	$\frac{1}{4R \times RC} = \frac{1}{1} \times \frac{208}{52}$		
Way 2	A complete method for the area include	2 4 7	M1	
Way 2 6(c)		Area = $\frac{52}{3}$ or exact equivalent e.g. $17\frac{1}{3}$ or 17.3 recurring (i.e. a clear	M1	
•	A complete method for the area include and BC using their values.	ling correct attempts at finding AB  Area = $\frac{52}{3}$ or exact equivalent e.g.		
•	A complete method for the area include and BC using their values.	Area = $\frac{52}{3}$ or exact equivalent e.g. $17\frac{1}{3}$ or 17.3 recurring (i.e. a clear	A1	
6(c)	A complete method for the area include and <i>BC</i> using their values. $= \frac{52}{3}$	ling correct attempts at finding AB  Area = $\frac{52}{3}$ or exact equivalent e.g.  17 $\frac{1}{3}$ or 17.3 recurring (i.e. a clear dot over the 3)  Attempts to find C using $y = 0$ in	A1 (3)	
•	A complete method for the area include and <i>BC</i> using their values. $= \frac{52}{3}$ $y = 0 \Rightarrow 2x - 14 = 0 \Rightarrow x = 7$	Area = $\frac{52}{3}$ or exact equivalent e.g. $17\frac{1}{3}$ or 17.3 recurring (i.e. a clear dot over the 3)  Attempts to find $C$ using $y = 0$ in the equation obtained in part (b)  Uses shoelace method. Must see a	A1 (3) M1	
6(c) Way 3	A complete method for the area include and BC using their values. $= \frac{52}{3}$ $y = 0 \Rightarrow 2x - 14 = 0 \Rightarrow x = 7$ $\frac{1}{2} \begin{vmatrix} 1 & 7 & -\frac{5}{3} & 1 \\ 4 & 0 & 0 & 4 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} -\frac{20}{3} - 28 \end{vmatrix}$	ling correct attempts at finding AB  Area = $\frac{52}{3}$ or exact equivalent e.g.  17 $\frac{1}{3}$ or 17.3 recurring (i.e. a clear dot over the 3)  Attempts to find C using $y = 0$ in the equation obtained in part (b)  Uses shoelace method. Must see a correct method including $\frac{1}{2}$ .  Area = $\frac{52}{3}$ or exact equivalent e.g.  17 $\frac{1}{3}$ or 17.3 recurring (i.e. a clear	A1 (3) M1 M1	

Way 4 6(c)	$y = 0 \Rightarrow 2x - 14 = 0 \Rightarrow x = 7$	Attempts to find $C$ using $y = 0$ in the equation obtained in part (b)	M1
	$\int_{-\frac{5}{3}}^{1} \left( \frac{3x}{2} + \frac{5}{2} \right) dx +$	$\int_{1}^{7} \left( -\frac{2x}{3} + \frac{14}{3} \right) \mathrm{d}x$	
	$= \left[\frac{3x^2}{4} + \frac{5}{2}x\right]_{-\frac{5}{3}}^{1}$	$+\left[-\frac{2x^2}{6} + \frac{14}{3}x\right]_1^7$	M1
	$= \left(\frac{3}{4} + \frac{5}{2}\right) - \left(\frac{75}{36} - \frac{25}{6}\right) +$	$\left(-\frac{49}{3} + \frac{98}{3}\right) - \left(-\frac{1}{3} + \frac{14}{3}\right)$	1411
		tes with correct integration on $l_1$ and	
	<u> </u>	ren line between their -5/3 and 1 and $l_2$ between 1 and their 7.	
	$=\frac{52}{3}$	Area = $\frac{52}{3}$ or exact equivalent e.g. 17 $\frac{1}{3}$ or 17.3 <b>recurring (i.e. a clear</b>	A1
	3	dot over the 3)	
			(3)

	PhysicsAndMa	athsTutor.com	<u> </u>
Question Number	Sch	eme	Marks
7 (i)	$\frac{2+4x^3}{x^2} = \frac{2}{x^2} + 4x = 2x^{-2} + 4x$	Attempts to split the fraction. This can be awarded for $\frac{2}{x^2}$ or $\frac{4x^3}{x^2}$ or may be implied by the sight of one correct index e.g $px^{-2}$ or $qx$ providing one of these terms is obtained correctly. So for example $\frac{2+4x^3}{x^2} = 2+4x^3+x^{-2}$ would be M0 as the $x^{-2}$ has been obtained incorrectly.	M1
	$\int 2x^{-2} + 4x  dx = 2 \times \frac{x^{-1}}{-1} + 4 \times \frac{x^{2}}{2} (+c)$	dM1: $x^n  o x^{n+1}$ on any term. <b>Dependent</b> on the first M. A1: At least one term correct, simplified or un-simplified. Allow powers <b>and</b> coefficients to be unsimplified e.g. $2  imes \frac{x^{-2+1}}{-1}$ , $+4  imes \frac{x^{1+1}}{2}$	dM1A1
	$=-\frac{2}{x}+2x^2+c$	All correct and simplified including the + c. Accept equivalents such as $-2x^{-1} + 2x^2 + c$	A1
			(4)
	There are no marks in (ii) for use integr		
(ii)	$\int \left(\frac{4}{\sqrt{x}} + k\right) dx$ $= \int (4x^{-0.5} + k) dx = 4\frac{x^{0.5}}{0.5} + kx(+c)$	M1: Integrates to obtain either $\alpha x^{0.5}$ or $kx$ A1: Correct integration (simplified or un-simplified). Allow powers and coefficients to be un-simplified e.g. $4\frac{x^{-0.5+1}}{0.5}$ . There is no need for $+c$	M1A1
	$\left[4\frac{x^{0.5}}{0.5} + kx\right]_{2}^{4} = 30 \Rightarrow$ Substitutes both $x = 4$ and $x = 2$ in subtracts either way rou Condone poor use or omission	and and sets equal to 30	M1
	$2k + 16 - 8\sqrt{2} = 30 \implies k = 7 + 4\sqrt{2}$	ddM1: Attempts to solve for $k$ from a linear equation in $k$ . Dependent upon both M's and need to have seen $\int k  dx \rightarrow kx$ .  A1: $7 + 4\sqrt{2}$ or exact equivalent e.g. $7 + 2^{2.5}$ , $7 + 4 \times 2^{0.5}$	<b>dd</b> M1A1
			(9 marks)
			() marks)

Question Number	Sche	eme	Marks
8(a)	$f(3) = 2(3)^{3} - 5(3)^{2} - 23(3) - 10$ or $2x^{2} + \dots$ $x - 3)2x^{3} - 5x^{2} - 23x - 10$	Attempts to calculate $f(\pm 3)$ or divides by $(x-3)$ . For long division need to see minimum as shown with a constant remainder.	M1
	(Remainder =) -70	-70	A1
			(2)
(b)	Mark (b) and	(c) together	
	$f(-2) = 2(-2)^{3} - 5(-2)^{2} - 23(-2) - 10$ Or $2x^{2} + \dots$ $x + 2 \overline{\smash)2x^{3} - 5x^{2} - 23x - 10}$	Attempts $f(\pm 2)$ or divides by $(x+2)$ . For long division need to see minimum as shown with a constant remainder.	M1
	Remainder = 0, hence $x + 2$ is a factor	Obtains a remainder zero and makes a conclusion (not just a tick or e.g. QED). Do not need to refer to the remainder in the conclusion but a zero remainder must have been obtained. (May be seen in a preamble)	A1*
	Note that <b>just</b> $f(-2) = 0$ therefore $(x + 2)$		
	be some evidence	of a calculation	(2)
(c)	$\frac{2x^3 - 5x^2 - 23x - 10}{(x+2)} = ax^2 + bx + c$	Divides $f(x)$ by $(x+2)$ or compares coefficients or uses inspection to obtain a quadratic expression with $2x^2$ as the first term.	M1
	$2x^2 - 9x - 5$	Correct quadratic seen	A1
	dM1: Attempt to factorise their 3TQ (2 $2x^2 - 9x - 5$ is factorised as $(x - 5)(x + \frac{1}{2})$ appears A1: $(x+2)(2x+1)(x-5)$ oe e.g. $2(x+2)$ one line. Must appear here and not in (d	$(x^2)$ . The usual rules apply here so if $(x^2)$ , this scores M0 unless the factor of 2 slater. $(x^2)(x+\frac{1}{2})(x-5)$ . All factors together on	dM1A1
	SC: This is a hence question but we candidates in this part who use their grant of the company o	will allow a special case of 1100 for raphical calculators to get roots of -2,	

Question Number	Scheme	Marks
	But note that if all that is seen is $(x+2)(x+\frac{1}{2})(x-5)$ this scores 1000	
		(4)

(d)	$3^t = '5' \Rightarrow t \log 3 = \log'5'$	Solves $3^t = k$ where $k > 0$ and follows from their (c) to obtain $t \log 3 = \log k$ .  Accept sight of $t = \log_3 k$ where $k > 0$ and follows from their (c)	M1
	$\Rightarrow t = \text{awrt } 1.465 \text{ only}$	t = awrt 1.465 and no other solutions	A1
			(2)
			(10 marks)

Question Number	Scl	neme	Marks
9(a)	$f(x) = 8x^{-1} + \frac{1}{2}x - 5$ $\Rightarrow f'(x) = -8x^{-2} + \frac{1}{2}$	M1: $-8x^{-2}$ or $\frac{1}{2}$ A1: Fully correct $f'(x) = -8x^{-2} + \frac{1}{2}$ (may be un-simplified)	M1A1
	Sets $-8x^{-2} + \frac{1}{2} = 0 \Rightarrow x = 4$	M1: Sets their f'(x) = 0 i.e. a "changed" function (may be implied by their work) and proceeds to find x. A1: $x = 4$ (Allow $x = \pm 4$ )	M1A1
	(4,-1)	Correct coordinates (allow $x = 4$ , $y = -1$ ). Ignore their (-4,)	A1
			(5)
(b)(i)	(x=)2, 8	x = 2 and $x = 8$ only. Do not accept as coordinates here.	B1
(b)(ii)	(4, 1)	(4, 1) or follow through on their solution in (a). Accept $(x, y+2)$ from their $(x, y)$ . With no other points.	B1ft
(b)(iii)	$(x=)2,\frac{1}{2}$	Both answers are needed and <b>accept</b> $(2, 0), (\frac{1}{2}, 0)$ here. Ignore any reference to the image of the turning point.	B1
			(3)
			(8 marks)

Question Number	Scheme		Marks
	Mark (a) and	(b) together	
10(a)	$(1+ax)^{20} = 1^{20} + {}^{20}C_{1}1^{1}$	$^{9}(ax)^{1} + ^{20}C_{2}1^{18}(ax)^{2}$ .	
	Note that the notation $\begin{pmatrix} 20 \\ 1 \end{pmatrix}$	may be seen for ${}^{20}C_1$ etc.	
	$\int_{0}^{20} C_1 1^{19} (ax)^1 = 4x \Rightarrow 20a = 4 \Rightarrow a = 0.2$	M1: Uses either ${}^{20}C_1(1^{19})(ax)^1 = 4x^1$ or $20a = 4$ to obtain a value for $a$ .	M1A1
		A1: $a = 0.2$ or equivalent	
			(2)
<b>(b)</b>		Uses ${}^{20}C_{2}(1^{18})(ax)^{2} = px^{2}$ and their	
	$\Rightarrow \frac{20 \times 19}{2} \times ('0.2')^2 = px^2$	value of $a$ to find a value for $p$ . Condone the use of $a$ rather than $a^2$ in finding $p$ . Maybe implied by an attempt to find a value for $190a^2$ or	M1
	$\Rightarrow p = \dots$	190a. Note: ${}^{20}C_{18}$ can be used for ${}^{20}C_2$	
	p = 7.6	Accept equivalents such as $\frac{38}{5}, \frac{190}{25}$	A1
			(2)
(c)	Term is ${}^{20}C_4 1^{16} (ax)^4 \Rightarrow q =$	Identifies the correct term and uses their value of $a$ to find a value for $q$ . Condone the use of $a$ rather than $a^4$ . Must be an attempt to calculate ${}^{20}C_4a^4$ or ${}^{20}C_4a$ or ${}^{20}C_{16}a^4$ or ${}^{20}C_{16}a$	M1
	$q = {}^{20}C_4 \times 0.2^4 = \frac{969}{125} = (7.752)$	$q = \frac{969}{125}$ or exact equivalent e.g. $7.752, 7\frac{94}{125}$ . $q = \frac{969}{125}x^4$ scores A0 but $qx^4 = \frac{969}{125}x^4$ scores A1.	A1
			(2)
			(6 marks)

· .	PnysicsAndiviaths Lutor.com	
Question Number	Scheme	Marks
11(i)	$3\cos^2 x + 1 = 4(1 - \cos^2 x)$ or $3(1 - \sin^2 x) + 1 = 4\sin^2 x$ or $3 + \tan^2 x + 1 = 4\tan^2 x$ or $3\frac{\cos 2x + 1}{2} + 1 = 4\frac{1 - \cos 2x}{2}$ Uses $\sin^2 x = 1 - \cos^2 x$ to produce an equation in $\cos^2 x$ or uses $\cos^2 x = 1 - \sin^2 x \text{ to produce an equation in } \sin^2 x \text{ or uses}$ $\cos^2 x + \sin^2 x = 1 \text{ and divides by } \cos^2 x \text{ to produce an equation in } \tan^2 x \text{ or uses } \sin^2 x \text{ and } \cos^2 x \text{ in } \tan^2 x \text{ or uses } \sin^2 x \text{ and } \cos^2 x \text{ in } \tan^2 x \text{ or uses } \sin^2 x \text{ and } \cos^2 x \text{ in } \tan^2 x \text{ or uses}$	M1
	$\Rightarrow \cos^2 x = \frac{3}{7} \text{ or } \sin^2 x = \frac{4}{7} \text{ or}$ $\tan^2 x = \frac{4}{3} \text{ or } \cos 2x = -\frac{1}{7}$ Correct value for $\cos^2 x$ or $\sin^2 x$ or $\tan^2 x$ or $\cos 2x$ . This may be implied by $\cos x = \sqrt{\frac{3}{7}} \text{ or } \sin x = \sqrt{\frac{4}{7}} \text{ or } \tan x = \sqrt{\frac{4}{3}}$	A1
	$\Rightarrow \cos x = \pm \sqrt{\frac{3}{7}} \Rightarrow x = \cos^{-1}\left(\sqrt{\frac{3}{7}}\right)$ A correct order of operations to obtain a correct expression for $x$ . E.g. $\cos^2 x = p \Rightarrow \cos x = \sqrt{p} \Rightarrow x = \cos^{-1}\sqrt{p} \text{ or}$ $\sin^2 x = p \Rightarrow \sin x = \sqrt{p} \Rightarrow x = \sin^{-1}\sqrt{p} \text{ or}$ $\tan^2 x = p \Rightarrow \tan x = \sqrt{p} \Rightarrow x = \tan^{-1}\sqrt{p} \text{ or}$ $\cos 2x = p \Rightarrow 2x = \cos^{-1}p \Rightarrow x = \frac{1}{2}\cos^{-1}p$ This may be implied by one correct answer for their values.	
	$\Rightarrow x = \text{awrt } 0.86, 2.28, 4.00, 5.43$ $\Rightarrow x = \text{awrt } 0.86, 2.28, 4.00, 5.43$ A1: All four of awrt 0.86, 2.28, 4.00, 5.43 with no additional solutions in the range and ignore solutions outside the range.	A2,1,0
	Note that answers in degrees are: 49.11, 130.89, 229.11, 310.89 Allow A1 for awrt two of these but deduct the final A mark. For answers given as awrt $0.27\pi$ , $0.73\pi$ , $1.27\pi$ , $1.73\pi$ , allow A1 only for any 2 of these but deduct the final A mark.	
		(5)

(22)		atns I utor.com	
(ii)	$5\sin(\theta+10^\circ)=\cos(\theta+10^\circ)$	M1: Reaches $tan() = \alpha$ where $\alpha$ is a	
	$\Rightarrow \tan(\theta + 10^\circ) = 0.2$	constant including zero.	M1A1
	$\Rightarrow tar(0+10)=0.2$	A1: $\tan() = 0.2$	
		For the correct order of operations to	
	2 1 (0.2) 100	produce one value for $\theta$ .	
	$\Rightarrow \theta = \tan^{-1}(0.2) - 10^{\circ}$	Accept $\theta = \tan^{-1}(\alpha) - 10$ , $\alpha \neq 0$ or one	dM1
		correct answer as evidence.	
		<b>Dependent on the first M.</b> A1: One of awrt $\theta = 1.3, 181.3$	
	0 1010100	A1: Both awrt $\theta = 1.3, 181.3$ and no	-
	$\Rightarrow \theta = \text{awrt } 1.3^{\circ}, 181.3^{\circ}$	other solutions in range and ignore	A1A1
		solutions outside the range.	
		(ii) cannot score the final 2 A marks but	
	the earlier marks are av	vailable (maximum 11100)	(=)
			(5)
	Alternative 1 for	r (ii) by squaring:	(10 marks)
		i (ii) by squaring.	
	$5\sin() = \cos()$		
	$\Rightarrow 25\sin^2() = \cos^2()$	Squares both sides, replaces	
	$\Rightarrow 25(1-\cos^2()) = \cos^2()$	$\sin^2()$ by $1-\cos^2()$ or	
	or	replaces $\cos^2()$ by	M1
	$25\sin^2() = 1 - \sin^2()$	$1-\sin^2()$ and reaches	
		$\sin^2() = \text{ or } \cos^2() =$	
	Leading to $\sin^2() =$ or $\cos^2() =$		
	siii () – oi cos () –		
	1	Correct value for $\sin^2()$ or	
	$\sin^2() = \frac{1}{26}$	$\cos^2()$ . This may be implied	
	or	$b_{V}\sin(\cdot) = \frac{1}{2}$ or	A1
		by $\sin() = \frac{1}{\sqrt{26}}$ or $\cos() = \sqrt{\frac{25}{26}}$	7.11
	$\cos^2() = \frac{25}{26}$	225	
		$\cos(\ldots) = \sqrt{26}$	
	$\theta = \sin^{-1} \frac{1}{1} - 10^{\circ}$	For the correct order of	
	$\theta = \sin^{-1} \frac{1}{\sqrt{26}} - 10^{\circ}$	operations to produce one value	
	or	for $\theta$ as shown or accept one	<b>d</b> M1
	$\theta = \cos^{-1} \frac{5}{\sqrt{26}} - 10^{\circ}$	correct answer as evidence.	
	√26	Dependent on the first M.	
		A1: One of awrt $\theta = 1.3, 181.3$	
	$\Rightarrow \theta = 1.3^{\circ}, 181.3^{\circ}$	A1: Both awrt $\theta = 1.3, 181.3$ and	A1A1
	→ <i>v</i> −1.3 ,101.3	no other solutions in range and	AIAI
		ignore solutions outside the range.	
	Note that final answers in <b>radians</b> in	n (ii) cannot score the final 2 A marks	
		vailable (maximum 11100)	

	Alternative 2 for (ii) Using	the addition formulae	
Alt (ii)	$5\sin\theta\cos 10 + 5\cos\theta\sin 10 = \cos\theta\cos 10 - \sin\theta\sin 10$ Uses the correct addition formulae on both sides and rearranges to $\tan() =$		M1
	$\tan \theta = \frac{\cos 10 - 5\sin 10}{5\cos 10 + \sin 10} = (0.0229)$ Correct value for $\tan \theta$		A1
	$\tan \theta = 0.0229 \Rightarrow \theta = \dots$	Uses arctan to produce one value for $\theta$ . <b>Dependent on the first M.</b>	<b>d</b> M1
	$\Rightarrow \theta = 1.3^{\circ}, 181.3^{\circ}$	A1: One of awrt $\theta = 1.3, 181.3$ A1: Both awrt $\theta = 1.3, 181.3$ and no other solutions in range and ignore solutions outside the range.	A1A1
	Note that final answers in <b>radians</b> in (ii) cannot score the final 2 A marks but the earlier marks are available (maximum 11100)		
			(5)

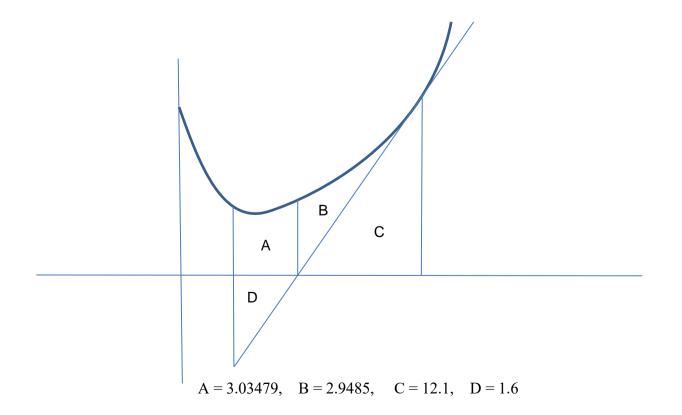
	PhysicsAndMathsTutor.com			
Question Number	Sche	me	Marks	
12(a)	$y = \frac{3}{4}x^2 - 4\sqrt{x} + 7 \Rightarrow \frac{dy}{dx} = \frac{3}{2}x - 2x^{-0.5}$	M1: Differentiates to obtain at least one correct power for one of the terms in $x$ . (may be un-simplified) e.g. $x^2 \to x^{2-1}$ or $\sqrt{x} \to x^{\frac{1}{2}-1}$ A1: Correct derivative. Allow unsimplified e.g. $2 \times \frac{3}{4} x^{2-1}$ or $-4 \times \frac{1}{2} x^{\frac{1}{2}-1}$	M1A1	
	At $x = 4$ $\frac{dy}{dx} = \frac{3}{2}(4) - 2(4)^{-0.5} =$	Substitutes $x = 4$ into a changed function in an attempt to find the gradient.	M1	
	$y-11 = "5"(x-4)$ or $y = mx+c \Rightarrow 11 = "5" \times 4 + c \Rightarrow c = \dots$	Correct straight line method using $(4, 11)$ correctly placed and their $dy/dx$ at $x = 4$ for the tangent <b>not the normal</b> . If using $y = mx + c$ , must reach as far as finding a value for $c$ . <b>Dependent on the previous M</b> .	<b>d</b> M1	
	y = 5x - 9	Correct printed equation with no errors seen. Beware of the "5" appearing from wrong working.	A1*	
	Following a correct derivative, if can is fine if they then complete c However, following a correct deriv dy/dx = 5 and then proceeds to obtain the final mark can be withheld. S candidate is considering	didate states $x = 4$ so $dy/dx = 5$ , this orrectly – allow full marks. Vative, if the candidate <u>just</u> states in the correct straight line equation, ome evidence is needed that the		
			(5)	

# For part (b), in all cases, look to apply the appropriate scheme that gives the candidate the best mark

	Finds area under curve between 1 and 4 and subtracts triangle C	
	(see diagram at end)	
(b) Way 1	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7  dx = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$ $M1: x^n \to x^{n+1} \text{ on any term.}$ May be un-simplified e.g. $x^2 \to x^{2+1}, x^{0.5} \to x^{0.5+1},$ $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3} \times \frac{3}{4}x^{2+1}$ , $-\frac{2}{3} \times 4x^{0.5+1}, 7x^1 \text{ and } + c \text{ is}$ not required.	M1A1
	Tangent meets $x$ axis at $x = 1.8$ This may be embedded within a triangle area below or may be seen on a diagram.	B1
	Area of triangle = $\frac{1}{2} \times (4 - 1.8') \times 11 = (12.1)$ Correct method for the area of a triangle - look for $\frac{1}{2} \times (4 - 1.8') \times 11$ This may be implied by the evaluation of $\int_{12.8'}^{4} 5x - 9  dx = \left[5\frac{x^2}{2} - 9x\right]_{18'}^{4}$	M1
	Correct method for area = Area $A$ + Area $B$ + Area $C$ - Area $C$ $\left(\frac{1}{4}4^3 - \frac{8}{3} \times 4^{1.5} + 7 \times 4\right) - \left(\frac{1}{4}1^3 - \frac{8}{3} \times 1^{1.5} + 7 \times 1\right) - 12.1'$ Correct combination of areas. <b>Dependent on both previous method marks.</b>	ddM1
	Area of $R = \text{awrt } 5.98$ or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
		(6)
		(11 marks)

	F'. 1 1		
	Finds area under curve between 1 and "1.8" and adds "line – curve" or		
	"curve – line" between "1	.8" and 4	
(b) Way 2	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7  dx = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n  o x^{n+1}$ on any term. May be un-simplified e.g. $x^2  o x^{2+1}$ , $x^{0.5}  o x^{0.5+1}$ , $7  o 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3} \times \frac{3}{4} x^{2+1}$ , $-\frac{2}{3} \times 4x^{0.5+1}$ , $7x^1$ and $+c$ is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be seen on a diagram.	B1
	Area between "1.8" and 4 =		
	$\pm \int_{1.8'}^{4} \left(\frac{3}{4}x^2 - 4\sqrt{x} + 7\right) - (5x - 9) dx = \pm \left[\frac{1}{4}\right]$ $= \frac{56}{3} - 15.7182 (= 2.9)$ Attempts to integrate "curve – line" or "line – "1.8" and 4 and subtr	485) - curve", substitute the limits	M1
	Correct method for area = Are		
	$\left( \left( \frac{1}{4} "1.8"^3 - \frac{8}{3} "1.8"^{1.5} + 7 \times "1.8" \right) - \left( \frac{1}{4} 1^3 - \frac{8}{3} 1^{1.5} + 7 \times 1 \right) + 2.9485' \right)$		ddM1
	Correct combination of areas. Dependent on both previous method marks.		
	= awrt 5.98	Area of $R = \text{awrt } 5.98 \text{ or}$ allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)

	Uses "line – curve" or "curve – line"	between 1 and 4 and subtracts triangle	
	below	x axis	
(b) Way 3	$\pm \left( \frac{3}{4}x^2 - 4\sqrt{x} + 7 - 5x + 9 \right)$	$= \pm \left(\frac{3}{4}x^2 - 4\sqrt{x} - 5x + 16\right)$	
	$\pm \int \frac{3}{4}x^2 - 4\sqrt{x} - 5x + 16  dx = \pm \left(\frac{1}{4}x^3 - \frac{8}{3}x^{1.5} - \frac{5x^2}{2}\right) + kx(+c)$		M1A1
	M1: $x^n \to x^{n+1}$ on any term. May be un	$x^2 \rightarrow x^{2+1}$ ,	WITAI
	$x^{0.5} \rightarrow x^{0.5+1}, x \rightarrow x^{1+1}, 16 \rightarrow 16x^{1}$ . If ter	rms are not collected when subtracting	
	then the same condition applies.		
	A1: Correct integration as shown. Ma	y be un-simplified for coefficients and	
	powers and $+ c$ is not required.		
	This may be embedded within a triangle area below or may be seen		7.1
	Tangent meets x axis at $x = 1.8$	B1	
	Area of triangle = $\frac{1}{2} \times (1)$	$ 1.8'-1\rangle \times  5\times 1-9  = (1.6)$	M1
	Correct method for the area of a trian	ngle - look for $\frac{1}{2} \times (1.8 - 1) \times  5 \times 1 - 9 $	IVII
	Correct method for area = Area	A + Area B + Area D - Area D	
	$\left( \left( \frac{1}{4} 4^3 - \frac{8}{3} 4^{1.5} - \frac{5 \times 4^2}{2} + 16 \times 4 \right) - \left( \frac{1}{4} 1^3 - \frac{8}{3} 1^{1.5} - \frac{5 \times 1^2}{2} + 16 \times 1 \right) - 1.6' \right)$		ddM1
	Correct combination of areas. Dependent on both previous method marks.		
		Area of $R = \text{awrt } 5.98 \text{ or allow the}$	
	= awrt 5.98 exact answer of $\frac{359}{60}$ or equivalent.		A1
			(6)



Question Number	Scheme			
13(a)(i)	$(0,c^2)$			
	(-c, 0) $(c, 0)$			
	(-c, 0) $(c, 0)$			
	Or shape anywhere but not			
	The maximum must be smooth and not form a point and the branches must not			
	clearly turn back in on themselves.			
	or	D1		
	A continuous graph passing through or touching at the points $(-c, 0)$ , $(c, 0)$ and	B1		
	$(0, c^2)$ . They can appear on their sketch or within the body of the script but			
	there must be a sketch. Allow these marked as $-c$ , $c$ and $c^2$ in the correct			
	places. Allow $(0, -c)$ , $(0, c)$ and $(c^2, 0)$ as long as they are marked in the correct			
	places. If there is any ambiguity, the sketch takes precedence.			
	A fully correct diagram with the curve in the correct position and the intercepts and shape as described above. The maximum must be on the <i>y</i> -axis	B1		
	and the branches must extend below the <i>x</i> -axis.	B1		
(a)(ii)	There must be a sketch to score any marks in (a)			
()(-)	↑ Shape. A positive cubic with only			
	one maximum and one minimum. The	B1		
	curve must be smooth at the maximum and	ы		
	at the minimum (not pointed).			
	A smooth curve that touches or meets the			
	x-axis at the origin and $(3c,0)$ in the			
	correct place and no other intersections.			
	The origin does not need to be marked but	D1		
	the $(3c,0)$ does. Allow $3c$ or $(0,3c)$ to be	B1		
	marked in the correct place. May appear on			
	their sketch or within the body of the script. If there is any ambiguity, the sketch			
	takes precedence.			
	Maximum at the origin (allow the	D1		
	maximum to form a point or cusp)	B1		
	There must be a sketch to score any marks in (a)	(5)		
<b>(b)</b>	Intersect when $x^{2}(x-3c) = c^{2} - x^{2} \Rightarrow x^{3} - 3cx^{2} = c^{2} - x^{2}$			
	Sets equations equal to each other and attempts to multiply out the bracket or	M1		
	vice versa			
	Collects to one side (may be implied),			
	factorises the $x^2$ terms and obtains printed answer with no errors. <b>There must be an</b>			
	meet meet at working.			
	$\Rightarrow x^3 + (1 - 3c)x^2 - c^2 = 0*$ Allow $x^3 + x^2(1 - 3c) - c^2 = 0$ or $0 = x^3 + (1 - 3c)x^2 - c^2$ or			
	· · · ·			
	$0 = x^3 + x^2(1 - 3c) - c^2$			
		(2)		

(c)	$8 + 4(1 - 3c) - c^2 = 0$	Substitutes $x = 2$ to give a <b>correct</b> unsimplified form of the equation.	M1
	$c^2 + 12c - 12 = 0$	Correct 3 term quadratic. Allow any equivalent form with the terms collected (may be implied)	A1
	$(c+6)^{2} - 36 - 12 = 0 \Rightarrow c = \dots$ or $c = \frac{-12 \pm \sqrt{12^{2} - 4 \times 1 \times (-12)}}{2}$	Solves their 3TQ by using the formula or completing the square <b>only</b> . This may be implied by a correct <b>exact</b> answer for their 3TQ. (May need to check)	M1
	$4\sqrt{3}-6$	$c = 4\sqrt{3} - 6$ or $c = -6 + 4\sqrt{3}$ only	A1
			(4)
			(11 marks)

	PhysicsAndMathsTutor.com				
Question Number	Scheme		Marks		
14 (a)	Allow the use of $S$ or $S_n$ throughout without penalty. $S = a + ar + ar^2 +ar^{n-1}$ and $rS = ar + ar^2 + ar^3 +ar^n$ There must be a minimum of '3' terms and must include the first and the $n$ th term. Condone for this mark only $S = a + ar + ar^2 +ar^n$ and $rS = ar + ar^2 + ar^3 +ar^{n+1}$ and allow commas instead of +'s but see note below.		M1		
	$S - rS = a - ar^n$	Subtracts either way around. As a special case allow $S - rS = a + ar^n$ . For this mark, their $S$ and their $rS$ must be different but it must be $S$ and $S$ they are considering with possible missing terms or slips.	M1		
	$\Rightarrow S(1-r) = a(1-r^n) \Rightarrow S = \frac{a(1-r^n)}{(1-r)}$	dM1: Dependent upon both previous M's. It is for taking out a common factor of S and achieving $S =$ A1*: Fully correct proof with no errors or omissions. The use of commas instead of +'s is an error. $S = \frac{a(r^n - 1)}{(r - 1)}$ without reaching the printed answer is A0	dM1A1*		
(a) Way	$S = \frac{\left(a + ar + ar^{2} + \dots + ar^{n-1}\right)\left(1 - r\right)}{1 - r}$	Gives a minimum of '3' terms and must include the first and the $n$ th and multiplies top and bottom by $1-r$	M1		
	$S = \frac{a + ar + ar^{2} + \dots + ar^{n-1} - ar - ar^{2} - ar^{n-1}}{1 - r}$	Expands the top with a minimum of '3' terms in each and must include the first and the $n$ th term	M1		
	$S = \frac{a(1-r^n)}{(1-r)}$	dM1: Dependent upon both previous M's. It is for taking out a common factor of $a$ on top and achieving $S =$ A1*: Fully correct proof with no errors or omissions. The use of commas instead of +'s is an error. $S = \frac{a(r^n - 1)}{(r - 1)}$ without reaching the printed answer is A0	dM1A1		

PnysicsAndWatns Lutor.com				
(b)	$U = 180 \times 0.93^n$ with $n = 4$ or 5	Attempts $U = 180 \times 0.93^n$ with $n = 4 \text{ or } 5$ . Accept $U = 167.4 \times 0.93^n$ with $n = 3 \text{ or } 4$ Allow 93% for 0.93	M1	
	$U_{5} = 180 \times (0.93)^{5} = 125.2 \text{ (litres)}$	Cso. Awrt 125.2	A1*	
	Allow 93% or 1 – 7% for 0.93			
			(2)	
(c)	Attempts $S_n = \frac{a(1-r^n)}{(1-r)}$ with any combination of: n = 20/21 $a = 180/167.4$ and $r = 0.93Allow 93% for 0.93$		M1	
	$S = \frac{167.4(1 - 0.93^{20})}{(1 - 0.93)} \text{ or } S = 180 \times \frac{0.93(1 - 0.93^{20})}{(1 - 0.93)}$			
	$S = \frac{180(1 - 0.93^{21})}{(1 - 0.93)} - 180$ A correct numerical expression for the sum (may be implied by awrt 1831) Allow 93% or 1 – 7% for 0.93		A1	
	1831 (litres)	1831 <b>only</b> (Ignore units). Do not isw here, so 1831 followed by 1831×20 = scores A0.	A1	
			(3)	
			(9 marks)	

# **Listing:**

(b)	Sight of awrt 180, 167, 156, 145, 135, 125	Starts with 180 and multiplies by 0.93 either 4 or 5 times showing each result at least to the nearest litre and chooses the 5 <sup>th</sup> or 6 <sup>th</sup> term	M1	
	$U_{5} = 125.2  (\text{litres})$	Must see all values accurate to 1dp: e.g. awrt 180, 167.4, 155.7, 144.8, (134.6 or 134.7), 125.2	A1*	
		•		(2)
(c)	Total = $180 \times 0.93 + 180 \times 0.93^2 + \dots + 180 \times 0.93^{19} + 180 \times 0.93^{20} = \dots$ Finds an expression for the sum of 20 or 21 terms		M1	
	All sums accurate to awrt 1dp 167.4+155.7+144.8+134.6+125.2+42.2  A correct numerical expression for the sum (may be implied by awrt 1831)			
	1831 (litres)	1831 <b>only</b> (Ignore units). Do not isw here, so 1831 followed by 1831×20 = scores A0.	A1	
				(3)

PhysicsAndMathsTutor.com				
Question Number	Scheme	Marks		
15	Area of triangle = $\frac{1}{2} \times (2r)^2 \sin\left(\frac{\pi}{3} \text{ or } 60\right)$ or $\frac{1}{2} \times (r)^2 \sin\left(\frac{\pi}{3} \text{ or } 60\right)$	M1		
	Correct method for the area of either triangle. Ignore any reference to which triangle they are finding the area of.	1111		
	Area of sector = $\frac{1}{2} \times r^2 \times \frac{\pi}{3}$ Use of the sector formula $\frac{1}{2}r^2\theta$ with $\theta = \frac{\pi}{3}$ which may be embedded	M1		
	within a segment			
	Area $R = \text{Sector} + 2 \text{ Segments} = \frac{1}{2}r^2 \times \frac{\pi}{3} + 2 \times \left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \times \frac{\sqrt{3}}{2}\right)$			
	Area $R = \text{Triangle} + 3 \text{ Segments} = \frac{1}{2}r^2 \times \frac{\sqrt{3}}{2} + 3 \times \left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \times \frac{\sqrt{3}}{2}\right)$ Area $R = 3 \text{ Sectors} - 2 \text{ Triangles} = 3 \times \frac{1}{2}r^2 \times \frac{\pi}{3} - 2 \times \left(\frac{1}{2}r^2 \times \frac{\sqrt{3}}{2}\right)$			
	Area $R = Big triangle - 3$ White bits	M1A1		
	$= \frac{1}{2} \times (2r)^2 \frac{\sqrt{3}}{2} - 3 \times \left(\frac{1}{2}r^2 \times \frac{\sqrt{3}}{2} - \left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \times \frac{\sqrt{3}}{2}\right)\right)$ M1: A fully correct method (may be implied by a final answer of awrt 0.705 $r^2$ ) A1: Correct exact expression - for this to be scored $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ must be seen			
	$= \frac{1}{2}\pi r^2 - \frac{\sqrt{3}}{2}r^2 = r^2 \left(\frac{1}{2}\pi - \frac{\sqrt{3}}{2}\right)$ Cso (Allow $\frac{r^2}{2}(\pi - \sqrt{3})$ or any exact equivalent with $r^2$ taken out as a common factor)	A1		

